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B.Sc. Part-II, Paper IV

Differential Equation (linear diff. eqn with constant coefficient)

Ex. 1. Solve $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$, where $x = 2$, $\frac{dx}{dt} = 0$ when $t = 0$

Soln. The auxiliary equation is $m^2 - 3m + 2 = 0$
$\Rightarrow (m-1)(m-2) = 0 \Rightarrow m = 1$ or 2

Hence, the general solution is
$x = c_1 e^t + c_2 e^{2t}$ ——— (1)

Now, we evaluate c_1 and c_2 with the given conditions.

When $t = 0$, we have $x = 2$, hence from (1)
$2 = c_1 + c_2$ ——— (2)

Now, differentiating (1) w.r.t. t, we get
$\frac{dx}{dt} = c_1 e^t + c_2 2e^{2t}$

putting $t = 0$, $\frac{dx}{dt} = 0$, we get
$0 = c_1 + 2c_2$ ——— (3)

Solving (2) and (3), we get $c_1 = 4$ and $c_2 = -2$

Hence putting $c_1 = 4$ and $c_2 = -2$ in (1), the required solution is $x = 4e^t - 2e^{2t}$.

Ex. 2. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

Soln. The auxiliary equation is $m^2 - 6m + 9 = 0$
$\Rightarrow (m-3)^2 = 0 \Rightarrow m = 3, 3$, That is, the root $m = 3$ is repeated twice

Hence the general solution is $y = (c_1 + c_2 x)e^{3x}$.

Ex 3. Show that if $\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$ and if $\theta = \alpha$ and $\frac{d\theta}{dt} = 0$ when $t = 0$, then $\theta = \alpha \cos(\sqrt{g/l}\, t)$.

Soln. The given equation can be written as

$(D^2 + \frac{g}{l})\theta = 0$ where $D = \frac{d}{dt}$.

$\Rightarrow (lD^2 + g)\theta = 0$. $\therefore$ Auxiliary equation is $lm^2 + g = 0$

$\Rightarrow m^2 = -g/l \Rightarrow m = \pm i\sqrt{g/l}$.

Hence the general solution of the given diff. eqn is $\theta = \{A \cos \sqrt{g/l}\, t + B \sin \sqrt{g/l}\, t\}$ ——— (1)

Now, when $t = 0, \theta = \alpha$, Therefore from (1)

$\alpha = A + B \times 0 \Rightarrow A = \alpha$.

Again, differentiating (1), w.r.t. θ, we get

$\frac{d\theta}{dt} = -A \sin \sqrt{g/l}\, t . \sqrt{g/l} + B \cos \sqrt{g/l}\, t . \sqrt{g/l}$

But when $t = 0$, $\frac{d\theta}{dt} = 0$

$\therefore 0 = 0 + B . \sqrt{g/l} \Rightarrow B = 0$

Hence Substituting $A = \alpha$ and $B = 0$, in (1), we get

$\theta = \alpha \cos \sqrt{g/l}\, t$. ; proved.

Ex. 4 - Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

Soln. The auxiliary equation is $m^2 + m + 1 = 0$

Solving this equation, we get

$m = \frac{-1 \pm \sqrt{1 - 4.1.1}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

Here, real part $= -\frac{1}{2}$ and imaginary part $= \frac{\sqrt{3}}{2}$

$\therefore$ The general solution is $y = e^{-\frac{1}{2}x}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right)$.